On the Transmission Effectiveness in Multi-Rate Power-Control Capable Wireless Networks

Luigi Iannone, Serge Fdida

Abstract— **A formalization of Transmission Effectiveness in Wireless Networks is proposed, taking in account both multi-rate and power-control capabilities. Furthermore, these are the only assumptions made on the hardware. Formalization remains abstract from any particular hardware, MAC, routing protocol used. It is possible to use our formalization as a metric or efficiency measurements.**

Keywords— **Effectiveness, Multi-Hop, Wireless, Ad-Hoc.**

I. Introduction

In the last years there was, and is still continuing, an increasing interest on wireless technology, due to the wireless hardware becoming every time cheaper. Two paradigms exist in the wireless environment: the *Cellular Architecture* and the *Wireless Routing Architecture*.

In Cellular Architecture, routing features are concentrated in base stations that controls a region called cell. Any subscriber willing to communicate with another subscriber must send data to the base station of the cell in which it is located, even if the destination is in the same cell. Subscribers have no routing features, which are instead concentrated in the base stations that also form the infrastructure of the network.

In the Wireless Routing Paradigm every subscriber device serves as both the broadband access device for that subscriber as well as part of the network infrastructure. Each subscriber automatically forward traffic to other subscribers as needed to ensure full and continuous network coverage and facilitate network growth. Wireless Routing Paradigm is quite general and accounts different solutions like Ad-Hoc networks, Sensor Network and Mesh Networks [1], [2]. Typically, Wireless Routing Networks self-configure themselves, and decrease the link distances needed for connectivity, permitting in this way a higher throughput. Current technology offers several solution to build such networks, like Hiperlan/2 [3], BlueTooth [4] and IEEE802.11 [5]. This latter has become the most famous one, due to its presence in several *"flavors"*, as long as the low cost of the basic solution IEEE802.11b.

On the other hand, several proposals of MAC protocols have been developed with the only purpose of optimizing the medium access. Both existing commercial solutions and theoretical MAC usually offer some common features, particularly the possibility to transmit at different rates, depending on channel conditions, distance, and transmitting power. The last parameter can also be controlled,but it must follow some constraints depending on local government's organizations.

On top of all these MAC protocols, commercial or not, a large number of routing protocols have been developed. In a first time, some routing protocols that optimize traditional metrics have been proposed. Solution that take advantage of more specific features of the network have also been designed. In this heterogeneous scenario, interaction of MAC protocols and Routing Protocols resolve in one action: packet transmission in the wireless environment. Is it possible to have an abstract means to say if a transmission is effective or not? Possibly independent of the particular MAC and Routing technologies that produced the transmission. That account also for the transmission's rate and the interference effects. Having, in this way, an abstract mean to compare performances offered by interaction of different MAC/Routing protocols mix under a fixed traffic matrix. In the following paragraphs we propose a metric and a method to obtain an evaluation of such effectiveness.

The rest of this paper is organized as follows: Section II introduces the concept of Effectiveness of wireless communication, Section III talks about Interference in radio networks, Section IV completes the definition of Effectiveness adding Interference. Finally in Section V we give a possible utilization of what we proposed and Section VI concludes the paper.

II. Effectiveness of Wireless Communications

In this paper we focus on the single channel wireless routing paradigm where the lacking of a central coordinator leads typically to a suboptimal efficiency of the network.

In the literature, there exist several different algorithms for power control and rate adaptation. Nevertheless these two topics are treated separately to maximize only some features of wireless network and not the global efficiency. Typically, power control algorithms aim at reducing energy consumption and at avoiding lowering the overall network performance [6]. Instead, rate control algorithms try to maximize the goodput, adapting rate to channel conditions [7]. To our best knowledge, a general formalization of the abstract concept of effectiveness in wireless transmission, on which other more specific metrics can be build, has

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not been produced yet.

The rate that can be sustained in a radio communication between two devices depends basically on the SIR (Signal to Interference Ratio, [8]) value on the receiver. If host i is transmitting to host j, this will measure a signal to interference ratio equal to:

$$SIR = \frac{a_{ij} P_i}{AWGN + \sum_{k \neq i,j} a_{kj} P_k} \quad (1)$$

Where AWGN is the Additive White Gaussian Noise, and factor a_{ij} is the attenuation of power between host i and host j. They depend on the distance, the environment, and possible obstacles. SIR also shows clearly how power used by each host, has a direct relation to the produced interference.

Higher transmission rates are more sensitive to noise and need a higher minimum SIR to receive correctly. SIR is a function of the transmission power of the sender, the distance between source and destination, the Gaussian White Noise, and the interference due to other nodes that transmit at the same time. Thus, the sustainable rate depends indirectly on all these parameters. Regarding the power control, a hardware that can vary the emitted power in a continuous way would be expensive and also difficult to manage. Usually, hardware interfaces use a limited number of possible power levels N_P in exponential succession, because of the propagation characteristic of radio signals.

Definition 1: The ordered set of possible power levels:

$$\Pi = \{P_{min} = P[0], P[1], \, ..P[N_P] = P_{max}\} \quad (2)$$

Our proposal tries to answer at the question: how can we find/define a function that describes the effectiveness of the behavior of a node in a wireless network? The list of the properties that such function must have is:

- *The single transmission should use the highest possible transmission rate.* The higher is the transmission rate, the higher is the throughput and the shorter is transmission, thus enabling other hosts to transmit.
- *The single transmission should send data as far as possible.* The use of *long hops* corresponds to the well known concept of shortest-path in a hop count sense. The only real metric used up to now in wireless networks.
- *The single transmission should interfere as less as possible with other nodes.* To transmit at high rates and over *long hops* an high power is necessary, but this leads to high level of interference, and therefore lower the performances.

The first two points of the above list, lead to a metric similar to the one introduced by Gupta and Kumar in [9]. Let E be the function Effectiveness, then a first simple definition can be given.

Definition 2: Effectiveness is:

$$E(R, d) = R \cdot d \qquad \left[\frac{bit \cdot meter}{second}\right] \quad (3)$$

Where R is the communication rate and d is the distance of receiver. The formula can be interpreted

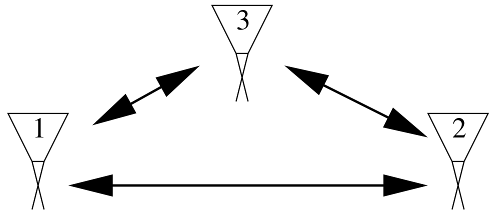

Fig. 1. Simply three nodes.

in this way: "effectiveness of a transmission is larger as more data (bits) are sent at a higher distance toward its destination (meter) in the same time unit (seconds)". Greater distances correspond to less hops thus the classic shortest-path. In figure 1 there is a simple scenario of three hosts[1], so close that the maximum transmission rate can be used. If host H_1 transmits to H_2, information is sent farther away than the case in which data are transmitted to H_3.

$$\begin{array}{c} E_{12} = (R_{12} \cdot d_{12}) > E_{13} = (R_{13} \cdot d_{13}) \\ where\ R_{12} = R_{13} \end{array} \quad (4)$$

What if we compare the same data sent to the same destination (i.e. the same distance) by different paths? Referencing the previous case we have:

$$E_{13,32} = \frac{(R_{13} \cdot d_{13}) + (R_{32} \cdot d_{32})}{2} \leq E_{12} = (R_{12} \cdot d_{12}) \quad (5)$$

This is true as long as the three hosts are so close that $R_{12} = R_{13} = R_{32}$ is a true relation and can be demonstrated easily by geometric construction. Equation (5) shows that if the transmissions can be done at the same rate, the most effective transmission is the direct one.

Of course, if the distances are so big that the sustainable rates are different on each link, it may happen that using shorter links, with higher transmission rate leads to higher effectiveness.

Insofar we did not take into account, while defining the function *Effectiveness*, the interference that a transmission causes on other nodes. The starting point is the following assumption: "The more interference is produced on other nodes, the more the average throughput (thus efficiency) of the network is lowered". And this assumption is demonstrated by simply considering the equation (1), the definition of SIR. In a first instance it is possible to think of a measure of the interference roughly as the power used to transmit, measured in mW. We will define in detail this function in the next Section, and we indicate it now only by I. The definition of Effectiveness can be so updated.

Definition 3: Effectiveness of a transmission is larger the more data (bits) are sent at higher distance toward its destination (meter) in the same time unit (seconds), the less interference (mW) is produced:

$$E(R, d, I) = \frac{R \cdot d}{I} \quad \left[\frac{bit \cdot meter}{second \cdot mW}\right] \quad (6)$$

[1]Hosts will be referred by letter H with a subscript throughout the paper.

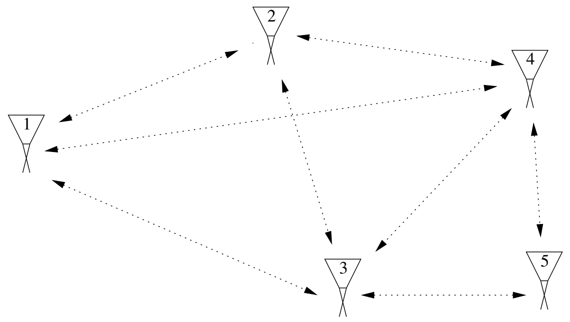

Fig. 2. Simple topology with five Radio Routers.

Returning to the three nodes topology, also if transmissions from H_1 to H_2 and H_3 can be done at the same maximum rate the latter can be performed using less transmitting power, so that:

$$E_{12} = \frac{(R_{12} \cdot d_{12})}{I_{12}} < E_{13} = \frac{(R_{13} \cdot d_{13})}{I_{13}} \quad (7)$$

where $R_{12} = R_{13}$, $I_{12} > I_{13}$, and I_{12} indicates the transmission power necessary to sustain rate R_{12} from H_1 to H_2. Note that equation (7) is not always true, strongly dependent on the used power [2].

To consider interference just proportional to the emitted power is too reductive.

Look at the scenario depicted in figure 2, and assume that H_3 is receiving data from H_5, which is transmitting with the lowest power level necessary. If H_1 has data to send to H_4 and the distance between them permits to transmit at maximum rate using high power radiation, H_3 probably will be disturbed and will not be able to correctly decode the H_5's transmission.

If otherwise H_1 decides to send data to H_4 multihopping them through H_2, and shrinks the transmitting power to the minimum necessary to perform a correct transmission to H_2 (and the same will do H_2 in forwarding to H_4), H_3 may receive its data undisturbed. Using the formula (6) introduced above:

$$E_{14} = \frac{R_{14} \cdot d_{14}}{I_{14}} < \frac{E_{12} + E_{24}}{2} = \frac{\frac{R_{12} \cdot d_{12}}{I_{12}} + \frac{R_{24} \cdot d_{24}}{I_{24}}}{2} \quad (8)$$

We are supposing $R_{12} = R_{24} = R_{14}$, thus for the equation (8) to be true, we must have $I_{14} > I_{12}$ and $I_{14} > I_{24}$. If we consider the interference as the transmission power, inequality (8) is true. But if H_3 and H_5 are not communicating, and H_1 transmits to H_4 at maximum rate with maximum transmission power, any host will be disturbed. Thus, still assuming that $R_{12} = R_{24} = R_{14}$, in formula what we want in the case that no node is disturbed, is:

$$E_{14} = \frac{R_{14} \cdot d_{14}}{I_{14}} > \frac{E_{12} + E_{24}}{2} = \frac{\frac{R_{12} \cdot d_{12}}{I_{12}} + \frac{R_{24} \cdot d_{24}}{I_{24}}}{2} \quad (9)$$

[2] Remember that transmission range and transmission power have not a linear relation ([8]). To double the transmission range, it is necessary to at least multiply the power by 4, depending on the environment.

This implies that the function interference $I(\cdot)$ has to describe both cases, while the simple assumption that $I = (Emitted\,Power)$ does not.

What comes out is that, *Interference* is not a function of only the emitted power but it should take in account also the state of the neighborhood, so to really give a measure of the perturbation that a transmission introduces in the network.

III. Interference

The real grade of *Interference* produced depends on neighborhood state, thus we formalize neighborhood condition in the next Subsection deferring the detailed definition of *Interference* to Subsection III-B.

A. Neighborhood

Usually in wireless environment the *neighborhood* of a node is the set of all other wireless nodes that have a direct link to that node. In a wireless context *"direct link"* means that two radio devices can hear each other, performing data exchange, except for temporary fading effects. This definition works well when only one transmission power level is used. Indeed, neighbors can be easily recognized, all nodes to which one node can talk are neighbors, the remaining are not. But some issues arise in radio interfaces that can transmit at different power levels. When a node that is able to talk to some other nodes using a certain power level shrinks the power, some hosts may not be reachable anymore.

Are this momentarily unreachable nodes still neighbors? Are they to be considered in routing decisions?

The same questions arise about the sustainable rate if devices are multi-rate capable. A node that is reachable with a certain power using a certain rate does not imply that it is possible to exchange data using a higher rate at the same power level.

To avoid misunderstandings and ambiguities a more detailed definition of neighborhood is necessary in the context of radio devices capable of power-control and multi-rate transmissions.

Definition 4: The total number of neighbors is the set of all nodes reachable with the basic (slowest) transmission rate using the highest transmission power (P_{max}). Call this set $\mathbb{N}_i$ for host H_i:

$$\mathbb{N}_i = \{All\,nodes\,reach.\,by\,H_i\,at\,basic\,rate\,and\,P_{max}\} \quad (10)$$

The above definition is the largest set of neighbors that can be defined, which turns out to assure the same connectivity as the one offered at PHY-Layer. A different definition would open the possibilities of scenarios where, even if the radio interface is able to communicate to some nodes by poor quality (slow) links (i.e. PHY-Layer connectivity exists), there is lack of connectivity at routing level due to an empty neighborhood set.

A difference can be introduced between the *Neighborhood* and the set of nodes that are reachable using a different transmitting power level.

Definition 5: Let H_i be a transmitting host, using a certain power P, the set of all nodes that can be contacted at basic rate by H_i are called the concerned neighbors*:*

$$\mathbb{N}_i\left(P\right) = \{\forall r \in \mathbb{N}_i \mid r\, reach.\, by\, H_i\, at\, basic\, rate\, and\, P\} \quad (11)$$

The defined notation is not ambiguous. Indeed, $\mathbb{N}_i$ without any following symbol indicate the *neighborhood* of host H_i. While $\mathbb{N}_i$ followed by the function operator $(\cdot)$ indicates the subset of the whole neighborhood $\mathbb{N}_i$ that forms the set of *concerned neighbors* related to the power level (P) within the function operator.

The wireless medium is very noisy, all MAC protocols typically use an active, per packet, acknowledgement mechanism. The aim of this choice is to recover as fast as possible packets that get lost. As a consequence, when two hosts communicate, both have to transmit a packet (DATA/ACK). Thus, both source/destination of the one-hop communication in turn have to transmit a packet using the same power level. Indeed, in this Section we always use the term *communication* instead of "the transmitting host" or "the receiving host". This is because no matter in which direction data flows, both communicating hosts have to send at least one packet.

Definition 6: Power level used in a communication between two hosts, H_i and H_j, is the same for both hosts and is indicated by $P_{ij}\,(= P_{ji})$.

This is not a bad assumption. Almost the totally of MAC Protocols rely on this assumption. Moreover, if the two communicating nodes use the same radio interface (as usually is the case), propagation differences in opposite direction are negligibles. In Figure 3 depicts an example of the above scenario.

Since the set of *concerned neighbors* contains all nodes that are reachable with a certain power level, these are also the nodes that have to refrain to transmit while H_i and H_j are communicating otherwise a collision will occur. In figure 3 all hosts that are in P_i or P_j transmission range are concerned neighbors of at least one of the two communicating hosts.

Definition 7: The set $\mathbb{R}_{ij}\left(P_{ij}\right)$ of all hosts that have to remain quiet during the communication of H_i and H_j, transmitting with power level P_{ij} is:

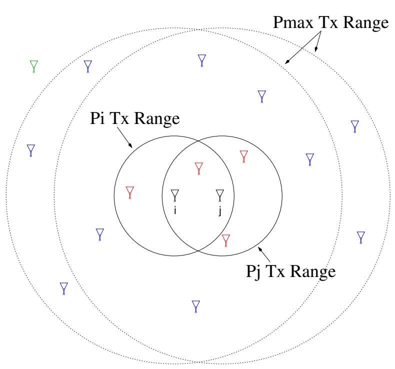

Fig. 3. Example of two hosts communicating and their neighborhood.

$$\mathbb{R}_{ij}\left(P_{ij}\right) = \left[\left(\mathbb{N}_i\left(P_{ij}\right) \setminus H_j\right) \cup \left(\mathbb{N}_j\left(P_{ij}\right) \setminus H_i\right)\right] \quad (12)$$

Another important set is the one that groups all remaining nodes.

Definition 8: $\mathbb{C}_{ij}\left(P_{ij}\right)$ indicates the nodes that are in the neighborhood of at least one of the two communicating hosts but are not concerned neighbors:

$$\mathbb{C}_{ij}\left(P_{ij}\right) = \left[\left(\mathbb{N}_i \cup \mathbb{N}_j\right) \setminus \left(H_i \cup H_j \cup \mathbb{R}_{ij}\left(P_{ij}\right)\right)\right] \quad (13)$$

The last defined set of hosts has a fundamental importance to increase the wireless network efficiency. Indeed, in the above set, hosts are able to communicate to other hosts as long as their communication does not collide/interfere with the occurring data exchanges. Suppose that other two host need to communicate. Say host H_k needs to communicate with host H_h using P_{kh} transmission power. The communication is possible without collisions only if the following relations hold:

- $\mathbb{R}_{kh}\left(P_{kh}\right) \cap H_i = \emptyset$
- $\mathbb{R}_{kh}\left(P_{kh}\right) \cap H_j = \emptyset$
- $\mathbb{R}_{ij}\left(P_{ij}\right) \cap H_k = \emptyset$
- $\mathbb{R}_{ij}\left(P_{ij}\right) \cap H_h = \emptyset$

The set of concerned neighbors of a couple of communicating hosts must not contain neither the sender nor the receiver of another communicating couples.

Should be noted that to allow concurrent communication, whether the intersection of the sets of concerned neighbors of both pairs is or is not empty does not matter, as long as the previous relations hold.

The possibility to perform a concurrent communication is a function of the power level used. Indeed, suppose that $H_k \subseteq \mathbb{N}_i$.

In this case, relation $\mathbb{R}_{kh}\left(P_{max}\right) \cap H_i \neq \emptyset$ is always true, which means that H_k cannot communicate with P_{max} power.

But, if it shrinks the power to a lower value P_{kh}, the communication may occur. Figure 4 shows how also if $\mathbb{R}_{kh}\left(P_{kh}\right) \cap \mathbb{R}_{ij}\left(P_{ij}\right) \neq \emptyset$, i.e. the sets of concerned neighbors of the two pairs is not empty, communication may occur.

At this point it is possible to generalize the set of relations listed above.

Definition 9: Let $\mathrm{T_i}$ be the set of all nodes of the neighborhood of host H_i that are communicating:

$$\mathrm{T_i} = \{\forall\, \mathrm{H_r} \in \mathbb{N}_\mathrm{i} \mid \mathrm{H_r\ is\ communicating}\} \quad (14)$$

If two hosts H_i and H_j need to communicate with power P_{ij} the following relation must hold:

$$\mathbb{R}_{ij}\left(P_{ij}\right) \cap \left[T_i \cup T_j\right] = \emptyset \quad (15)$$

B. Interference formalization

Suppose that H_i and H_j are communicating with power P_{ij}. The set $\mathbb{R}_{ij}\left(P_{ij}\right)$ are all the hosts whose interference will result in a collision, while the set $\mathbb{C}_{ij}\left(P_{ij}\right)$ are all hosts that, like the rest of the network, will sense interference as a reduction of their SIR, according to the distance.

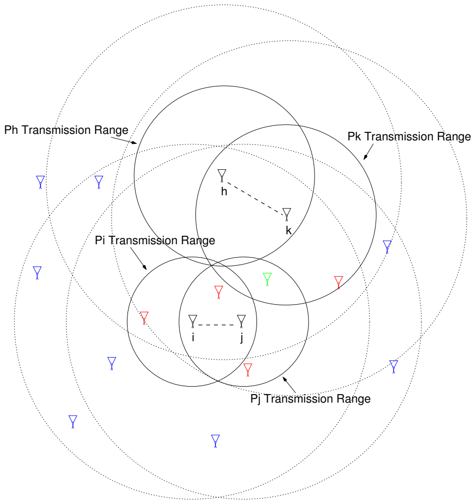

Fig. 4. Parallel communication occurs shrinking the used power.

Communication between H_i and H_j has also another side effect on hosts in $\mathbb{C}_{ij}(P_{ij})$. Members of this set can communicate but cannot use any power level they like. Thus, when a host is willing to communicate it is likely that to avoid collisions, according to its neighbors conditions, it can use only a subset of the set Π of all possible power levels.

Definition 10: Π_i is the subset of power levels that can be used by H_i without causing a collision:

$$\Pi_i = \{\forall P_i \in \Pi \mid \mathbb{N}_i(P_i) \cap T_i = \emptyset\} \tag{16}$$

Clearly, if a node communicates using a power level that is not in the Π_i set (i.e. a higher power), this results in collisions. The notation introduced is not ambiguous. The character Π without any subscript represents all possible power levels available physically on the interface. While character Π_i , with a subscript, represents all the power levels that H_i can use without causing collisions in a certain moment.

Definition 11: Interference caused by host H_i that transmits with power P_i is:[3]

$$I_i(P_i) = \begin{cases} P_i \cdot \frac{|\mathbb{N}_i(P_i)|}{|\mathbb{N}_i(\max(\Pi_i))|} & if\ P_i \geq \max(\Pi_i) \\ \sqrt{\frac{(\max(\Pi_i))^2 + (P_i)^2}{2}} & else \end{cases} \tag{17}$$

The above definition describes well the different grades of interference, expressed in mW, that a different power level creates, while abstracting from the physical phenomenon. Let's see how the just defined function behaves:

Case 1: $P_i \geq \max(\Pi_i)$
In this case, the used power level is higher than the maximum power that does not cause collisions. Therefore collisions are generated. Thus interference has a higher impact than what simply the power P_i can express. Because $|\mathbb{N}_i(P_i)| \geq |\mathbb{N}_i(\max(\Pi_i))|$ we obtain that $I_i(P_i) \geq P_i$ (depending on the topology) expressing well the negative effect of a transmission with such level of power. Furthermore, $|\mathbb{N}_i(P_i)| \geq [|\mathbb{N}_i(\max(\Pi_i))| + |\mathbb{N}_i(P_i) \cap T_i|]$, thus our definition is pessimistic, because accounts also nodes for which no collision occurs. Indeed, there are nodes that become *concerned neighbors,* as defined in section III-A, therefore not able to communicate.

[3]Operator $|\cdot|$ expresses the cardinality of a set.

Case 2: $P_i < \max(\Pi_i)$
In this case, the used power level is lower than the maximum possible without collisions. Therefore, this reduces the number of concerned neighbors, but this may reduce also transmission's rate and range, thus effectiveness. There is no gain in this reduction, that's why the function $I_i(P_i) = \sqrt{\frac{(\max(\Pi_i))^2+(P_i)^2}{2}}$ that decays slower than the power is introduced. This is a good property, because as we will see while computing effectiveness, if we fall in the present case interference has a minor role. Moreover the given definition works well in algorithms for energy consumption. This second property, while important, is not demonstrated here, due to lack of space.

Definition (11) describes the interference that a single node going to communicate will cause. But, in any communication two nodes are involved, each one sending a packet, as explained before. That's why we add the following:

Definition 12: If hosts H_i and H_j have to communicate, the overall interference produced is:

$$I_{ij}(P_{ij}) = I_i(P_i) + I_j(P_j) \tag{18}$$

As discussed previously we assume symmetric channel, thus $P_{ij} = P_i = P_j$.

The symbol I with only one subscript refers to the interference generated by one single node transmitting, while the symbol I with two subscripts refers to the overall interference generated by two communicating hosts.

One last observation can be done about the defined *Interference*. The given definition do not rely on the power attenuation factors. This has the nice consequence, that equation (17) is valid for any modulation technique. Indeed, changing modulation causes a change in the attenuation factors a_{ij} of the SIR formula (1). But, since these factors are not included in our definition, independency from modulation scheme is achieved.

IV. Effectiveness and Interference

In Section II it has been defined the effectiveness of a transmission as $E(R, d, I) = \frac{R \cdot d}{I}$, while definition of interference I was provided only in Section III-B. In this section we will see how *Interference* as defined by us fits well in the definition of *Effectiveness.* To accomplish this task we re-examine in this section the scenario of figure 2.

Suppose that H_5 is communicating with H_3 with power P_{53} and suppose that it is the minimum to perform the transmission at maximum rate. With this power level there are no other concerned neighbors (i.e. $\mathbb{R}_{53}(P_{53}) = \emptyset$).

If host H_1 wishes to communicate to H_4 what is likely to happen is that the direct communication will result in collision at least on H_3, because $\mathbb{R}_{14}(P_{14}) = \{H_2, H_3\}$ and $T_1 = \{H_3\}$, thus $\mathbb{R}_{14}(P_{14}) \cap T_1 \neq \emptyset$.

But if host H_1 multi-hops through host H_2, communicating to it with a power level $P_{12} = \max(\Pi_1)$, $P_{12} < P_{14}$, now $\mathbb{R}_{12}(P_{12}) = \emptyset$, thus $\mathbb{R}_{12}(P_{12}) \cap T_1 = \emptyset$.

The interference for the direct communication is:

$$\begin{aligned} I_1(P_{14}) &= P_{14} \cdot \tfrac{|\mathbb{N}_1(P_{14})|}{|\mathbb{N}_1(P_{12})|} = P_{14} \cdot \tfrac{3}{1} = 3 \cdot P_{14} \\ I_4(P_{41}) &= P_{41} \cdot \tfrac{|\mathbb{N}_4(P_{41})|}{|\mathbb{N}_4(P_{42})|} = P_{41} \cdot \tfrac{4}{1} = 4 \cdot P_{41} \\ I_{14}(P_{14}) &= I_1(P_{14}) + I_4(P_{41}) = 7 \cdot P_{14} \end{aligned} \tag{19}$$

Note that the produced interference is more than just the sum of the two used power levels due to collision that will occur on H_3. Multi-hopping through H_2 will generate the following interference:

$$\begin{aligned} I_1(P_{12}) &= P_{12} \cdot \tfrac{|\mathbb{N}_1(P_{12})|}{|\mathbb{N}_1(P_{12})|} = P_{12} \cdot \tfrac{1}{1} = P_{12} \\ I_2(P_{21}) &= P_{21} \cdot \tfrac{|\mathbb{N}_2(P_{21})|}{|\mathbb{N}_2(P_{21})|} = P_{21} \cdot \tfrac{2}{2} = P_{21} \\ I_{12}(P_{12}) &= I_1(P_{12}) + I_2(P_{21}) = 2 \cdot P_{12} \end{aligned} \tag{20}$$

Suppose that subsequent communication between H_2 and H_4 occurs while H_3 and H_5 are still communicating, using a power level $P_{24} = \max(\Pi_2)$. For the second-hop communication the interference will be:

$$\begin{aligned} I_2(P_{24}) &= P_{24} \cdot \tfrac{|\mathbb{N}_2(P_{24})|}{|\mathbb{N}_2(P_{24})|} = P_{24} \cdot \tfrac{2}{2} = P_{24} \\ I_4(P_{42}) &= P_{42} \cdot \tfrac{|\mathbb{N}_4(P_{42})|}{|\mathbb{N}_4(P_{42})|} = P_{42} \cdot \tfrac{1}{1} = P_{42} \\ I_{24}(P_{24}) &= I_2(P_{24}) + I_4(P_{42}) = 2 \cdot P_{24} \end{aligned} \tag{21}$$

We now can calculate the *Effectiveness* [4] in both cases expliciting the interference:

$$\begin{aligned} E_{12,24} &= \tfrac{E_{12}+E_{24}}{2} = \frac{\frac{R \cdot d_{12}}{I_{12}} + \frac{R \cdot d_{24}}{I_{24}}}{2} = \tfrac{R \cdot d_{12}}{2 \cdot I_{12}} + \tfrac{R \cdot d_{24}}{2 \cdot I_{24}} \\ E_{14} &= \tfrac{R \cdot d_{14}}{I_{14}} \end{aligned} \tag{22}$$

If host H_2 is almost in the middle between H_1 and H_4, then $d_{12} \approx d_{24} \approx \frac{d_{14}}{2}$ and $P_{12} \approx P_{24}$, consequently $I_{12} \approx I_{24}$, obtaining $E_{12,24} \approx \frac{R \cdot d_{14}}{2 \cdot I_{12}}$. Being $P_{12} < P_{14}$ it is also true that $2 \cdot P_{12} < 7 \cdot P_{14}$ and this clearly leads to affirm that:

$$E_{12,24} \approx \frac{R \cdot d_{14}}{2 \cdot P_{12}} > \frac{R \cdot d_{14}}{7 \cdot P_{14}} = E_{14} \tag{23}$$

What we can conclude is that multi-hop in this case is more effective because no critical interferences will occur.

Moreover, also in the simple topology of figure 2 there are cases where multi-hopping joined power-control lead to more than one communication occurring at the same time, and this can be considered as a better global network efficiency and is well expressed by our definition.

If hosts H_5 and H_3 are not communicating function *Effectiveness* have still to express what is the best behavior. In this second case T_1 is empty thus also $\mathbb{R}_{14}(P_{14}) \cap T_1$ is empty, no collision occurs and the single-hop communication is the most effective. Obviously now we have that $P_{14} = \max(\Pi_1)$, while P_{12} is just the minimum communication power to support maximum rate between hosts H_1 and H_2, therefore still $P_{12} < P_{14}$. The remaining sets, values and assumption are unchanged.

The *Interference* in the single-hop communication is:

$$\begin{aligned} I_1(P_{14}) &= P_{14} \cdot \tfrac{|\mathbb{N}_1(P_{14})|}{|\mathbb{N}_1(P_{14})|} = P_{14} \cdot \tfrac{3}{3} = P_{14} \\ I_4(P_{41}) &= P_{41} \cdot \tfrac{|\mathbb{N}_4(P_{41})|}{|\mathbb{N}_4(P_{41})|} = P_{14} \cdot \tfrac{4}{4} = P_{14} \\ I_{14}(P_{14}) &= I_1(P_{14}) + I_4(P_{41}) = 2 \cdot P_{14} \end{aligned} \tag{24}$$

We can already note that the *Interference* value has significantly lowered. For the two-hop communication we obtain:

$$\begin{aligned} I_{12}(P_{12}) &= I_1(P_{12}) + I_2(P_{21}) = 2 \cdot \sqrt{\tfrac{(P_{14})^2 + (P_{12})^2}{2}} \\ I_{24}(P_{42}) &= I_2(P_{24}) + I_4(P_{42}) = 2 \cdot \sqrt{\tfrac{(P_{14})^2 + (P_{24})^2}{2}} \end{aligned} \tag{25}$$

As can be observed interference seems augmented, how can this be explained? From the previous case the value $\max(\Pi_i)$ has changed, this is due to a difference in the neighborhood state, consequently also the grade of interference changes. The two-hop communication still uses the same power level as before. The change in the *Interference* value of the two-hop communication leads to an interesting result, multi-hop is not anymore the effective solution.

Indeed, comparing once again the two-hop communication, with lower power level, to the single-hop communication at higher power level, the effectiveness for the two cases is:

$$\begin{aligned} E_{12,24} &= \frac{\frac{R \cdot d_{12}}{I_{12}} + \frac{R \cdot d_{24}}{I_{24}}}{2} = \tfrac{R \cdot d_{12}}{2 \cdot I_{12}} + \tfrac{R \cdot d_{24}}{2 \cdot I_{24}} \approx \tfrac{R \cdot d_{14}}{2 \cdot I_{12}} \\ E_{14} &= \tfrac{R_{14} \cdot d_{14}}{I_{14}} \end{aligned} \tag{26}$$

What rest is to show that $E_{12,24} < E_{14}$. To do this it is sufficient to proof that $I_{14} < 2 \cdot I_{12}$. We can express I_{14} in a different form:

$$I_{14} = 2 \cdot P_{14} = 2 \cdot \sqrt{2} \cdot \sqrt{\frac{(P_{14})^2}{2}} \tag{27}$$

now it is easy to recognize that the following inequalities are true

$$2 \cdot \sqrt{2} \cdot \sqrt{\tfrac{(P_{14})^2}{2}} < 4 \cdot \sqrt{\tfrac{(P_{14})^2}{2}} < 4 \cdot \sqrt{\tfrac{(P_{14})^2 + (P_{12})^2}{2}} \tag{28}$$

But the quantity on the right is equal to $2 \cdot I_{12}$, proving that in absence of critical interferences (collision) the one-hop solution is more effective. Demonstration that the previous relation is true whatever is the position of H_2 in respect of the other two hosts, as long as $d_{14} \leq d_{12} + d_{24} \leq 2 \cdot d_{14}$, is omitted.

[4] Note that a particular value of the rate R is not specified because we are supposing that all communications are occurring at the same maximum rate. Thus, $R_{12} = R_{24} = R_{14}$, and is just expressed by the letter R. The aim of this supposition is to point out the behavior of the effectiveness when only the interference changes.

V. Effectiveness as metric of global efficiency

Our proposition may find several utilizations typically as dynamic metric in multi-path routing or as basic measure of efficiency in wireless networks. When a packet needs to be transmitted three parameters have to be set up by routing algorithm and MAC protocol:

- *Next-Hop:* A node possibly closer to the destination should be chosen.
- *Power:* The more power is used, the more resources of the medium are utilized (and subtracted to other nodes).
- *Rate:* Transmission rate should be chosen accordingly to the environment conditions (SIR) and transmission power.

All the three parameters are bound together by the Effectiveness metric. These parameters can be set accordingly to the destination of the packet with the aim to maximize the *Effectiveness* of the transmission of each packet. The final target is to optimize the global network behavior.

Good routing decision in wireless networks are a dynamic tradeoff between shortest-path, transmission power, produced interference, and transmission rate. The *Effectiveness* can be used to make routing decision.

Consider a network with N_H hosts $H_1, H_2, ..., H_{N_H}$. Let $S\left(t\right)$ be the transmission scheme at time t. The transmission scheme at a given time consists of all communicating node pairs at that time and, for each of these pairs, the transmission rate and the transmission power. Efficiency of the network at a given time can be defined as the sum of the *Effectiveness* of each single transmission in the network at that time:

$$Efficiency\left(t\right) = E\left(S\left(t\right)\right) \tag{29}$$

This notation expresses the summation of the *Effectiveness* of every ongoing *communication* of the transmission scheme, where *communication* means packet exchange. For example in 802.11 a *communication* is a RTS/CTS/DATA/ACK packets exchange.

We can extend this to an observation window obtaining our efficiency parameter:

$$Efficiency = \frac{1}{T}\int_0^T E\left(S\left(t\right)\right) \cdot dt \tag{30}$$

This definition may enable to compare the behavior of a network with a fixed matrix of traffic but using different routing policies. But, since this definition of efficiency leads to a value that can grow large depending on the traffic matrix, only comparison are possible and not absolute measures.

Since in the Wireless Routing Paradigm control is distributed, nodes are aware only of the direct neighbors state. Thus if *Effectiveness* is used to perform routing decision, this does not guarantees that the optimum of *Efficiency* is achieved. Where with *optimum* we indicate what the routing would be if arriving time of all packets is known a priori along with the global condition of the whole network at each time. Thus, the routing algorithm would be performed a priori offline.

On the other hand, the use of *Effectiveness* in routing decision will enable to achieve higher efficiency than other routing protocols that do not consider power control, rate adaptation, and interference estimation in their forwarding decisions.

VI. Conclusions

We give, in the previous paragraphs, a definition of Effectiveness of wireless transmission. What we propose is defined formally and abstracts from particular MAC and Routing protocols as long as the modulation scheme used. Also, the notion of Interference has been abstracted from physical phenomenon to a more general, but formal, definition. Work are in progress with the aim of use actively Effectiveness. This work emphasizes on the importance of cross-layer integration.

Two are the main work direction, the first is to extend the simulator ns-2 so to be able to calculate Effectiveness in Wireless simulation and efficiency of the whole network. This is also useful to enforce by some practical results our proposition. The second work direction is to use Effectiveness as a local metric in each node performing routing decisions.

References

[1] HTTP://www.ietf.org/html.charters/manet-charter.html

[2] D.Beyer, Fundamental characteristics and benefits of Wireless Routing (“Mesh”) Networks, Wireless Communications Association International, Technical Symposium, January 2002.

[3] ETSI, Broadband Radio Access Networks, Hiperlan Type 2 System Overview, February 2000.

[4] HTTP://www.Bluetooth.org, *Bluetooth V1.1 Core Specification*, 2003.

[5] IEEE, Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY)specifications; IEEE Standard 802.11, June 1999.

[6] A. Muqattash, M. Krunz, *Power Controlled Dual Channel (PCDC) Medium Access Protocol for Wireless Ad Hoc Networks*, Infocom 2003.

[7] W.H. Yuen, H. Lee, T.D. Anderson, *A Simple and Effective Cross Layer Networking System for Mobile Ad Hoc Networks*, IEEE PIMRC, September 2002.

[8] P. Mohana Shankar, *Introduction to Wireless Systems*, Wiley & Sons Editions, 2002.

[9] P. Gupta, P.R. Kumar, The Capacity of Wireless Networks, IEEE Trans. Inf. Theory, vol. 46 n. 2 pp 388-404, March 2000.